Enumeration of Polyabolos Constructible with Tangram and Sei Shonagon Chie no Ita

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Silhouette puzzles represent a classic category of geometric tiling problems where a specific set of *pieces*, must be arranged to form a target silhouette without overlapping. The most representative examples are *Tangram* (Figure 1), originated in China (first documented in 1813), and *Sei Shonagon Chie no Ita* (Figure 2), originated in Japan (documented in 1742) [1]. Despite their distinct historical origins, they share many properties. Both consist of a square dissected into seven pieces and the sum of the constituent edge lengths is also the same (20 edges of length 1 and 10 edges of length $\sqrt{2}$).

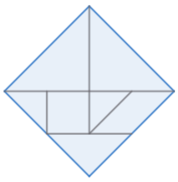

Figure 1: Tangram

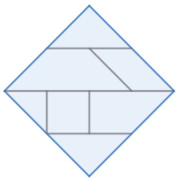

Figure 2: Sei Shonagon Chie no Ita

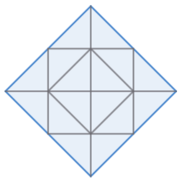

Figure 3: Polyabolo of area 16

However, a discrepancy in their combinatorial expressiveness has not yet quantified. Previous results considered only convex polygons. Wang and Hsiung [2] proved that Tangram can form 13 different convex polygons, while Fox-Epstein et al. [3] demonstrated that Sei Shonagon Chie no Ita can form 16. We extended the comparison to *polyabolos*, which are obtained by connecting by a set of congruent isosceles right triangles in edge-to-edge style. In Figure 3, we show an example of a polyabolo of size 16. It is easy to see that every piece of Tangram and Sei Shonagon Chie no Ita can be represented by a polyabolo of this scale. By exhaustive computational enumeration, we elucidate the total numbers of polyabolos of both puzzles. We consider two shapes are identical if they coincide after rotation or reflection. We adopted a "connectivity-based sequential construction" method. This functions as a search tree that starts with an arbitrary fixed initial piece and sequentially adds remaining pieces to the existing shape according to edge-adjacency rules. The details can be found in [4].

We performed the comparative enumeration for the two target puzzles:

- **Tangram:** The enumeration yielded a total of **5,583,516** distinct constructible polyabolos.
- **Sei Shonagon Chie no Ita:** The enumeration yielded a total of **10,889,227** distinct constructible polyabolos.

These results quantitatively prove that Sei Shonagon Chie no Ita possesses approximately 1.95 times the generative capacity of Tangram. While the pieces look similar, Sei Shonagon Chie no Ita exhibits a drastically higher level of geometric expressiveness when constraints are lifted to allow general non-convex shapes. All generated datasets and a custom web-based viewer have been made publicly available at `https://leeono8.github.io/polyabolo-viewer/`.

To understand the geometric origins of this discrepancy, we extended our analysis to seven additional puzzle variants, including those proposed by Fox-Epstein et al. and novel designs. By correlating the piece characteristics with the enumeration results, we identified three primary geometric factors governing expressiveness:

Large-Area Pieces: There is a negative correlation between the presence of large monolithic pieces and the number of configurations. Tangram contains two large triangles (area 4). Puzzles that replace these with smaller components (areas ≤ 3) showed significantly higher. Large pieces lack the flexibility to fill fine gaps or form complex local boundaries.

Piece Shape Diversity: Higher diversity in the shapes of constituent pieces reduces symmetry-induced redundancies. Diverse pieces reduce the likelihood that swapping two pieces results in an identical geometric state.

Effectiveness of Asymmetry: Asymmetric pieces contribute more to expressiveness than symmetric ones. Comparing puzzles with identical area distributions, those containing asymmetric pieces (like parallelograms) outperformed those with symmetric pieces (like squares). Asymmetry allows a piece to take on distinct states via reflection (flipping), effectively increasing the degrees of freedom in combination. Sei Shonagon Chie no Ita benefits significantly from this factor compared to the more symmetric composition of Tangram.

References

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[3] E. Fox-Epstein, K. Katsumata, and R. Uehara. The convex configurations of "Sei Shonagon Chie no Ita," tangram, and other silhouette puzzles with seven pieces, *IEICE Trans. on Fund. of Electro., Comm. and Comp. Sci.*, vol. 99, no. 6, pp. 1084–1089 (2016)

[4] H. Li. Enumeration of Polyabolos Constructible with Tangram and Sei Shonagon Chie no Ita, Master thesis, JAIST, 2026.